Enrollment No: ______________________ Exam Seat No:______________________

C. U. SHAH UNIVERSITY

Winter Examination-2021

Subject Name: Complex Analysis

Subject Code: 4SC05COA1 **Branch: B.Sc. (Mathematics)**

Semester: 5 **Date: 16/12/2021** **Time: 11:00 To 02:00** **Marks: 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) A function $u(x, y)$ is said to be harmonic if and only if_____. 01

(a) $u_{xx} + u_{yy} = 0$ (b) $u_{xx} - u_{yy} = 0$ (c) $u_{xy} + u_{yx} = 0$ (d) None

b) What is the value of m for which $2x - x^2 + my^2$ is harmonic? 01

(a) 1 (b)-1 (c) 2 (d) -2

c) Polar form of Cauchy- Riemann equation is____ 01

(a) $u_r = rv_\theta$ and $v_r = -r\, u_\theta$ (b)$u_r = \frac{1}{r} v_\theta$ and $v_r = -\frac{1}{r} u_\theta$

(c)$u_r = \frac{1}{r} v_\theta$and $v_r = -r\, u_\theta$ (d) None of these

d) If $f(z) = \bar{z}$ then f is differentiable_______. 01

(a) nowhere (b) only at $z = 0$ (c) everywhere (d) only at $z = 1$

e) A function $f(z)$ is analytic if 01

(a) Real part of $f(z)$ is analytic (b) imaginary part of $f(z)$ is analytic

(c) both (a) and (b) (d) None of these

f) Simple curve is also called 01

(a) Multiple curve (b) Jordan curve (c) Integral curve (d) None

g) Which of the following are fixed points of $w = \frac{2z+6}{z+7}$. 01

(a) 2,3 (b) 1,5 (c) -6,1 (d) 1,4

h) Let $W = \frac{az+b}{cz+d}$ then W is mobius transformation if____. 01

(a) $ad - cb = 0$ (b) $ad - cb \neq 0$ (c) $ad + cb = 0$ (d) None

i) If $f(z) = z + \bar{z}$ then $f(z)$ is _____. 01

(a) Purely real (b) Purely imaginary (c) Zero (d) None

j) If $f(z) = u + iv$ is an analytic function of complex variable $z = x + iy$, and $v = xy$ then $u =$ ___ 02

(a) $x^2 + y^2$ (b) $x^2 - y^2$ (c) $\frac{1}{2}(x^2 + y^2)$ (d) $\frac{1}{2}(x^2 - y^2)$

k) Define: Analytic function 01

l) State Liouville's theorem. 02

Attempt any four questions from the Q-2 to Q-8

Q-2		**Attempt all questions**	**(14)**
	(a)	Show that $f(z) = \begin{cases} \frac{x^3(1+i)-y^3(1-i)}{x^2+y^2} & ; z \neq 0 \\ 0 & ; z = 0 \end{cases}$ is continuous at origin.	05
	(b)	Find analytic function $f(z) = u + iv$ such that $u - v = x + y$.	05
	(c)	Evaluate $\lim_{z\to -i} f(z)$, where $f(z) = \begin{cases} \frac{z^2+3iz-2}{z+i} & , z \neq -i \\ 5 & , z = -i \end{cases}$	04
Q-3		**Attempt all questions**	**(14)**
	(a)	Sate and prove Cauchy's integral formula.	07
	(b)	Show that $u = \cos x \cos hy$ is harmonic, also find its harmonic conjugate.	07
Q-4		**Attempt all questions**	**(14)**
	(a)	State and prove C-R equation in cartesian coordinates.	07
	(b)	Suppose $f(z) = u + iv, z_0 = x_0 + iy_0$ and $w_0 = u + iv$ then $\lim_{z\to z_0} f(z) = w_0$ if and only $\lim_{(x,y)\to(x_0,y_0)} u(x,y) = u_0$f and $\lim_{(x,y)\to(x_0,y_0)} v(x,y) = v_0$.	05
	(c)	If $u(x,y) = \frac{x(1+x)+y^2}{(1+x)^2+y^2}$, $v(x,y) = \frac{y}{(1+x)^2+y^2}$ then find $f(z)$ in terms of z.	02
Q-5		**Attempt all questions**	**(14)**
	(a)	Evaluate: $\int_C \frac{dz}{z^2+9}$ where $C: \lvert z \rvert = 5$.	05
	(b)	Evaluate $\int_C z^2\,dz$ where C is the path joining the points $z = 1 + i$ to $z = 2(1 + 2i)$ along the straight line joining $1 + i$ to $2(1 + 2i)$.	05
	(c)	State and prove ML- inequality.	04
Q-6		**Attempt all questions**	**(14)**
	(a)	State and prove Cauchy's inequality.	07
	(b)	State Cauchy-Goursat theorem and hence evaluate $\int_C \frac{z^3+z^2+z+1}{z(z-1)^2} dz,\ C: \lvert z \rvert \leq 2$.	07
Q-7		**Attempt all questions**	**(14)**
	(a)	Let $f(z) = u + iv$ be analytic in domain D then prove that real component u and imaginary component v are harmonic function.	05
	(b)	Prove that $\left\lvert \int_C \frac{\log z}{z} dz \right\rvert \leq 2\pi \left(\frac{\pi+\log R}{R}\right)$ where C is circle $\lvert z \rvert = R$.	05
	(c)	find $\int_C (\bar{z})^2\,dz$ where C is part of line which we can obtain from the point $z = 0$ to $z = 2 + i$.	04
Q-8		**Attempt all questions**	**(14)**
	(a)	Find the bilinear transformation which maps points $z = 0,1,\infty$ into $w = -5,-1,3$	05

respectively.

(b) Find image of $|z - 3i| = 3$ under the mapping $w = \frac{1}{z}$. 05

(c) Transform the curve $x^2 - y^2 = 4$ under the mapping $w = z^2$. 04